Everything you wanted to know about RCAT but were afraid to ask

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Abstract

Given a stochastic process, the dual reversed process is the one with the same state space, to which the direction of time is reversed. A process is reversible is the reversed process is stochastically identical to the original process. The theory of reversible processes is quite useful. In the first place, we know that if a Markov process is reversible than the reversed process enjoys the Markov property. Moreover, reversed processes are useful in calculating steady state distribution. That is, however, at the price that one knows the rates of both the forward and reversed processes. In some cases calculating the rate of the reverse process can be computationally expensive. One way to proceed in the calculation of the rates of the reversed process is to use Kolmogorov's criteria, which essentially states that the exit rates of each state is the same in both the reversed and the forward processes, and the product of the rates of any finite cycle-path has to be same regardless of whether one takes the forward direction or the backwards one. If the process has finite statespace, Kolgomorov's criteria can be very useful, yet computationally quite expensive. In case with infinite statespace, it is sometime possible to parameterise the cycles in order to calculate the rates.

How the theory of reversible process can be applied to stochastic process algebra (for example PEPA[2]) is an interesting area of research. Due to the compositional nature of process algebra, application of reversible processes is to provide means to derive the reversed process of a complicated Markov chain by looking at the subcomponents of whose reversed proceses are known. In other words, one decomposes Markov chains into smaller chains and calculates the reverse processes of each subchain, in order to obtain the reversed process of the original process. This has been essentially achieved by Harrison [1] in the theorem 'Reversed Compound Agent Theorem' (RCAT). We shall present RCAT in pure PEPA [2] style, show some simple applications to queueing networks. We shall suggest an extension of the theorem to deal with the extension of process algebra with immediate actions. We shall see that essentially we are going to apply the basic principle of reversible processes in a similar way as the original RCAT theorem, yet we need to get rid of the immediate actions.

References

[1] P.G. Harrison. Turning back time in Markovian process algebra. *Theoretical Computer Science*, 290:1947–1986, 2003.

[2] J. Hillston. *A Compositional Approach to Perfomance Modelling.* PhD thesis, Department of Computer Science, Edinburgh, 1994.

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