An attempt to give a clear semantics of the extension of PEPA for massively parallel processes

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Outline

Motivations

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Extensions of PEPA, massively parallel processes and mass actions

Semantics of this double extension

Conclusion

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2. To have a semantics that can be explained by the standard operational semantics of PEPA.

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Massively parallel processes

Defined in [Calter & al 2004, Benoit & al 2005, Hillston 2005].

- A syntactic shortcut to represent massive parallelism :

$$P[n] = \underbrace{P \| \ldots \| P}_{n}$$

$$P[a,b][n] = \underbrace{P \underset{a,b}{\bowtie} \ldots \underset{a,b}{\bowtie} P}_{n}$$

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- A method to derive a small set of ODEs that describes the evolution of the number of components in their different states.

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- $H \Rightarrow$ multiplication of mass actions.

$$R = r_1 \times r_2 \qquad r_\alpha(P \underset{\beta}{\overset{\alpha}{\bowtie}} Q) = r_\alpha(P) \times r_\alpha(Q)$$

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$$R = r_1 \times r_2 \qquad r_\alpha(P \underset{\beta}{\overset{\alpha}{\bowtie}} Q) = r_\alpha(P) \times r_\alpha(Q)$$

- L ⇒ standard operator, bounded capacity.

$$R = \frac{r_1}{r_\beta(P)} \times \frac{r_2}{r_\beta(Q)} \times min(r_\beta(P), r_\beta(Q))$$

$$r_\beta(P \underset{\beta}{\overset{\alpha}{\bowtie}} Q) = min(r_\beta(P), r_\beta(Q))$$

PEPA models of this double extension

1. a set of declarations, each noted :

$$C \stackrel{def}{=} S$$

with S a PEPA term designing a sequential process

$$S := (\alpha, r).S \mid S + S \mid C$$

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2. a root term :

$$P := P \underset{L}{\overset{H}{\bowtie}} P \mid P/L \mid (S_1[n_1]||\ldots||S_k[n_k])$$

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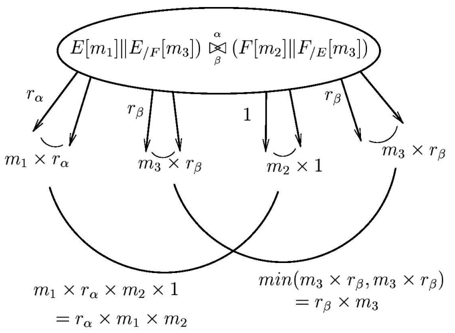

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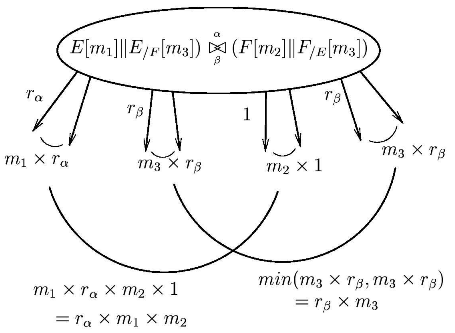

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Semantics of this double extension

- Decomposition of a process into a vector of sequential processes :

$$P = C[m_1] \underset{L}{\overset{H}{\bowtie}} C'[m_2]$$

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$$X_t = (X^1, \ldots, X^n) \qquad \text{where } n = m_1 + m_2$$

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- Let $B_t^{i,s}$ be a binary random process :

$$\forall i \in \{1, \ldots, n\},\ \forall s \in \mathbf{S} \qquad B_t^{i,s} = \begin{cases} 1 & \text{if } X_t^i = s \\ 0 & otherwise \end{cases}$$

Semantics of this double extension

- Let N_t^s be the random process giving the number of processes in state $s \in \mathbf{S}$ w.r.t. time :

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- The semantics is defined as the expectation of N_t^s :

$$\forall s \in \mathbf{S} \qquad \mathbb{E}[N_t^s]$$

1^{st} example : independent processes

Let P be the parallel product of identical independent processes with k states $\{s_1, \ldots, s_k\}$:

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- Evolution of the probability of one sequential process :

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{p}(t).\mathbf{Q} \qquad \mathbf{p}(0) = \pi$$

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- Evolution of the expectation of the number of processes :

$$\frac{d(\mathbb{E}[N_t^{s_1}], \ldots, \mathbb{E}[N_t^{s_k}])}{dt} = (\mathbb{E}[N_t^{s_1}], \ldots, \mathbb{E}[N_t^{s_k}]).\mathbf{Q}$$

$$(\mathbb{E}[N_t^{s_1}], \ldots, \mathbb{E}[N_t^{s_k}]) = (m_1, \ldots, m_k)$$

2^{nd} example : dependent processes

Let P be the PEPA model of
two processes, X^1 and X^2 :

$$X^1 : \quad \begin{array}{rcl} A & \stackrel{def}{=} & (\alpha, r_\alpha).A' \\ A' & \stackrel{def}{=} & (\tau, r_A).A \end{array}$$

$$X^2 : \quad \begin{array}{rcl} B & \stackrel{def}{=} & (\alpha, r_\alpha).B' \\ B' & \stackrel{def}{=} & (\tau, r_B).B \end{array}$$

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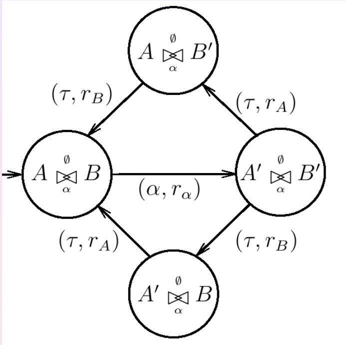

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- Expectations of the number of components in each state :

$$\mathbb{E}[N_t^A] = \mathbb{P}(X_t^1 = A) \qquad \mathbb{E}[N_t^B] = \mathbb{P}(X_t^2 = B)$$
$$\mathbb{E}[N_t^{A'}] = \mathbb{P}(X_t^1 = A') \qquad \mathbb{E}[N_t^{B'}] = \mathbb{P}(X_t^2 = B')$$

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- The following ODEs describe these expectations :

$$\frac{d\mathbb{E}[N_t^A]}{dt} = -r_\alpha \times \mathbb{P}(X_t = A \underset{\alpha}{\overset{\emptyset}{\bowtie}} B) + r_A \times \mathbb{E}[N_t^{A'}]$$
$$\frac{d\mathbb{E}[N_t^{A'}]}{dt} = r_\alpha \times \mathbb{P}(X_t = A \underset{\alpha}{\overset{\emptyset}{\bowtie}} B) - r_A \times \mathbb{E}[N_t^{A'}]$$
$$\frac{d\mathbb{E}[N_t^B]}{dt} = -r_\alpha \times \mathbb{P}(X_t = A \underset{\alpha}{\overset{\emptyset}{\bowtie}} B) + r_B \times \mathbb{E}[N_t^{B'}]$$
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- Expectations of the number of components in each state :

$$\begin{aligned} \mathbb{E}[N_t^A] &= \mathbb{P}(X_t^1 = A) & \mathbb{E}[N_t^B] &= \mathbb{P}(X_t^2 = B) \\ \mathbb{E}[N_t^{A'}] &= \mathbb{P}(X_t^1 = A') & \mathbb{E}[N_t^{B'}] &= \mathbb{P}(X_t^2 = B') \end{aligned}$$

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- We cannot get rid of $\mathbb{P}(X_t = A \underset{\alpha}{\overset{\emptyset}{\bowtie}} B)$.

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  1. to clarify when this semantics can be reduced as a small set of ODEs, when it cannot.
  2. to see in which cases this semantics can be approximated by a small set of ODEs. Maybe when the number of processes grows the previous semantics becomes a good approximation of this one?